Chapter 6

Random Variables

Identify the support and type of each random variable:

1. $X = \{\text{the number of heads in 10 flips of a fair coin}\}$
   - Support: $[0, 1, 2, \ldots, 10]$
   - Type: DISCRETE

2. $X = \{\text{the time between arrivals at a store}\}$
   - Support: $[0, \infty)$
   - Type: CONTINUOUS

   *THINK ABOUT FRACTIONS OF TIME: 1.123456 SECONDS

3. $X = \{\text{the number of cars sold in a day at a dealership}\}$
   - Support: $[0, 1, 2, \ldots]$
   - Type: DISCRETE

   *I CAN'T SELL HALF A CAR BUT I ALSO COULD SELL AN ARBITRARILY LARGE AMOUNT: COUNTABLE INFINITE

4. $X = \{\text{the number of defective items in a batch of 20}\}$
   - Support: $[0, 1, 2, \ldots, 20]$
   - Type: DISCRETE

5. $X = \{\text{the number of attempts required to pass an exam}\}$ *
   - Support: $[0, 1, 2, \ldots]$
   - Type: DISCRETE

6. $X = \{\text{the weight of randomly selected oranges}\}$ *
   - Support: *$[0, \infty)$
   - Type: CONTINUOUS

*CAN I HAVE NEGATIVE WEIGHT?

Discrete Probability Distributions

Problem 1 Let Y = the number of people waiting at a bus stop. The probability distribution of Y is given as follows:

y	0	1	2	3	4	5	6
$P(Y = y)$	0.3	0.25	0.2	0.1	0.05	0.05	0.05

TOTAL 1.00

Q1. Is this a legitimate probability distribution? YES

Q2. Find the probability that there are exactly 2 people waiting at the bus stop. 0.2

Q3. Find the probability that there are more than 3 people waiting at the bus stop. 0.15

Q4. $P(1 \leq Y \leq 4)$ 0.6

Q5. Find $\mathbb{E}Y$ $\mathbb{E}Y = \sum y\, P(Y=y) = 0(0.3)+1(0.25)+2(0.2)+3(0.1)+4(0.05)+5(0.05)+6(0.05) = 1.7$*

Q6. Find $VarY$ *

$VY = \sum (y-\mu)^2 P(Y=y)$

$= (0-1.7)^2(0.3)+(1-1.7)^2(0.25)+(2-1.7)^2(0.2)\ (3-1.7)^2(0.1)+(4-1.7)^2(0.05)+(5-1.7)^2(0.05)+(6-1.7)^2(0.05) = 2.91$

Problem 2 Let Z = the number of students waiting for office hours with a professor. The probability distribution of Z is given as follows:

z	0	1	2	3	4
$P(Z = z)$	0.5	0.2	0.15	0.1	0.05

TOTAL 1.00

Q1. Is this a legitimate probability distribution? YES

Q2. Find the probability that no students are waiting for office hours. 0.5

Q3. Find the probability that at least 2 students are waiting for office hours. $P(Z \geq 2) = 0.3$

Q4. $P(1 < Z \leq 4)$ 0.5

Q5. Find $\mathbb{E}Z$ $= 1$

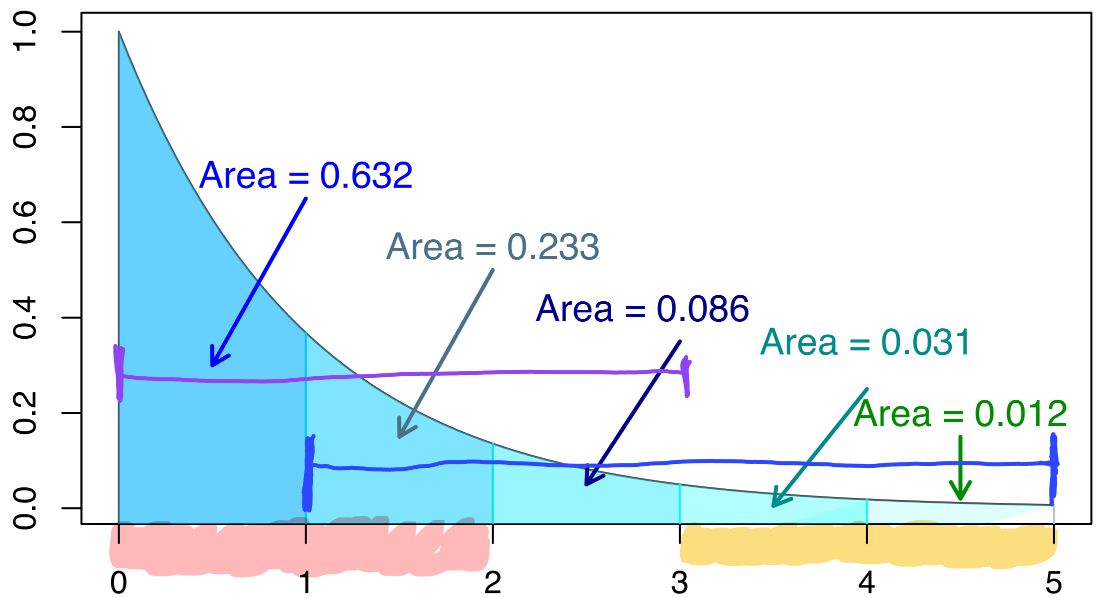
Q6. Find $VarZ$ $= 1.5$

Q7. Find σ_Z $= \sqrt{1.5} \approx 1.224745$

Chapter 7

Continuous Probability Distributions

Problem 1



REMEMBER THAT APPROXIMATIONS ARE IMPERFECT

1. What is the total area of this curve? 0.994 ≈ 1.00
2. What is the proportion of the population between 0 and 2? 0.865
3. What is the probability of any individual in this population being between 3 and 5? 0.043
4. What is the probability of any individual in this population being between 0 and 3? 0.951
5. What is the probability of any individual in this population being greater than 1? 0.362
6. $P(1 \leq x < 3)$ = 0.319
7. $P(1 < x \leq 5)$ = 0.362

ALL OUTCOMES ARE

Problem 2 The waiting time at a bus stop for the next bus to arrive is uniformly distributed between 0 and 12 minutes.

EQUALLY LIKELY

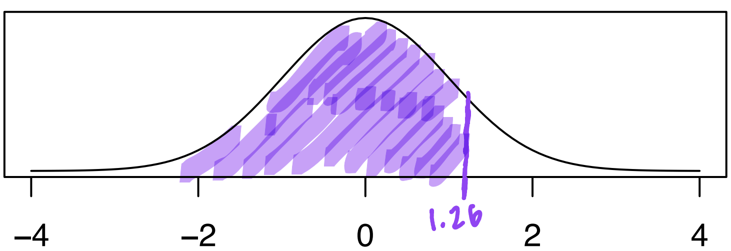
1. Find the probability that the waiting time is less than 4 minutes. $4/12$
2. Find the probability that the waiting time is greater than 7 minutes. $12-7/12 = 5/12$
3. Find the probability that the waiting time is between 5 and 10 minutes. $10-5/12 = 5/12$

Z-Tables

NOTE: ALL MY Z & t RESULTS WILL BE DIFFERENT FROM YOUR TABLE RESULTS BECAUSE I USED A COMPUTER

AS LONG AS THEY'RE CLOSE DON'T WORRY

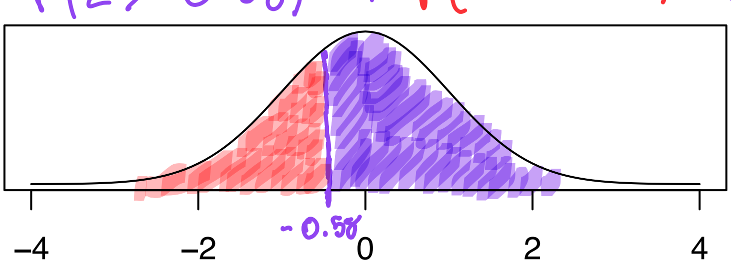
Problem 1 $P(Z < 1.26)$ (i.e., area to the left of $z = 1.26$);



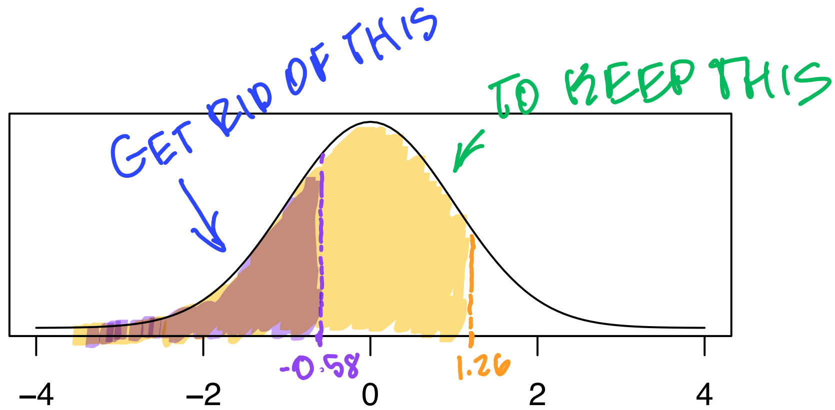
$P(Z<1.26) = 0.896165$

Problem 2 $P(Z > -0.58)$ (i.e., area to the right of $z = -0.58$);

$P(Z>-0.58) = 1-P(Z<-0.58) = 0.7190427$



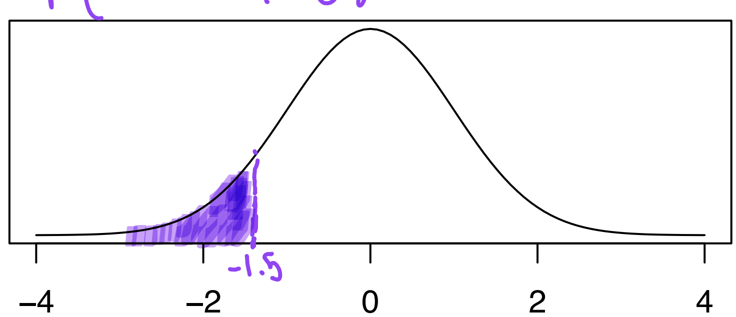
Problem 3 $P(-0.58 < Z < 1.26)$ (i.e., area between $z = -0.58$ and $z = 1.26$);



0.896165 − 0.2809573 = 0.615208

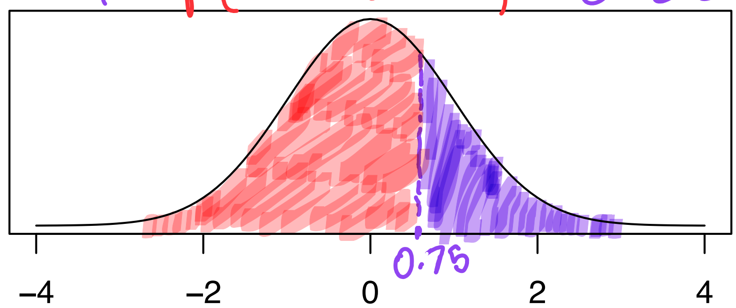
Problem 4 $P(Z < -1.5)$ (i.e., area to the left of $z = -1.5$);

$P(Z < -1.5) = 0.0668072$



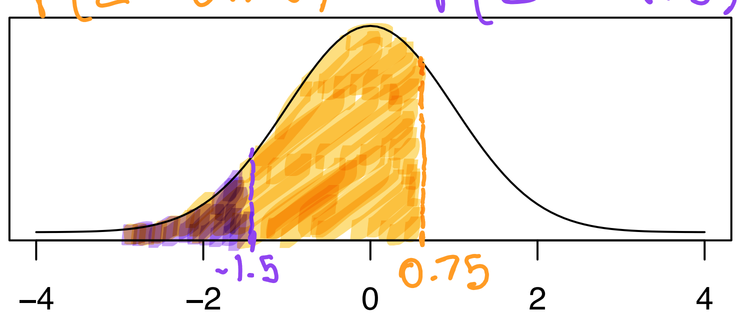
Problem 5 $P(Z > 0.75)$ (i.e., area to the right of $z = 0.75$);

$1 - P(Z < 0.75) = 0.2266274$



Problem 6 $P(-1.5 < Z < 0.75)$ (i.e., area between $z = -1.5$ and $z = 0.75$);

$P(Z < 0.75) - P(Z < -1.5) = 0.7333135$



Non-standard Normal Distributions

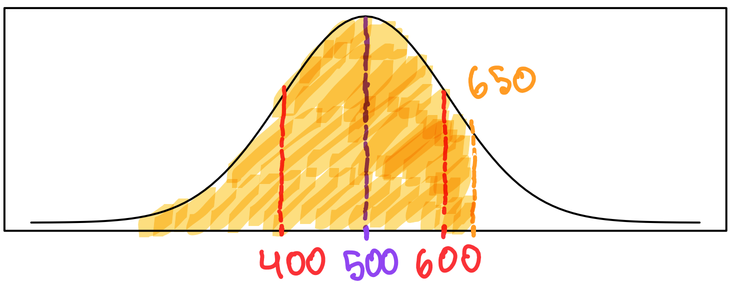
Problem 1 The SAT Math scores for a certain year were normally distributed with a mean of 500 and a standard deviation of 100.

$\mu = 500 \quad \sigma = 100 \qquad z = \frac{x - \mu}{\sigma}$

1. What proportion of students scored less than 650?

$x = 650$

$$z = \frac{650 - 500}{100} = 1.5$$



CHECK YOUR WORK
HOW MANY σ IS 650 FROM 500?

$$P(z < 1.5) = 0.9331928$$

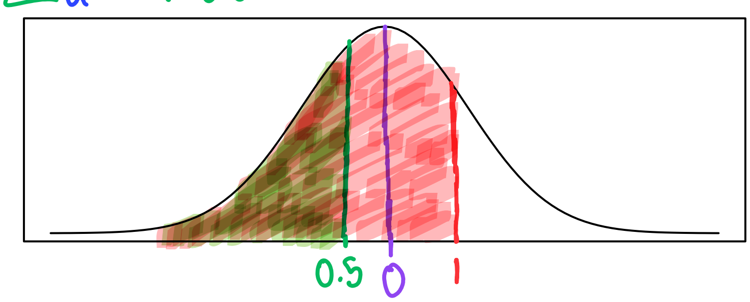
* EMPIRICAL RULE STATES HOW MANY σ 95% OF DATA IS?

2. What proportion of students scored between 450 and 600?

L U

$$z_L = \frac{450 - 500}{100} = 0.50$$

$$z_u = 1.00$$



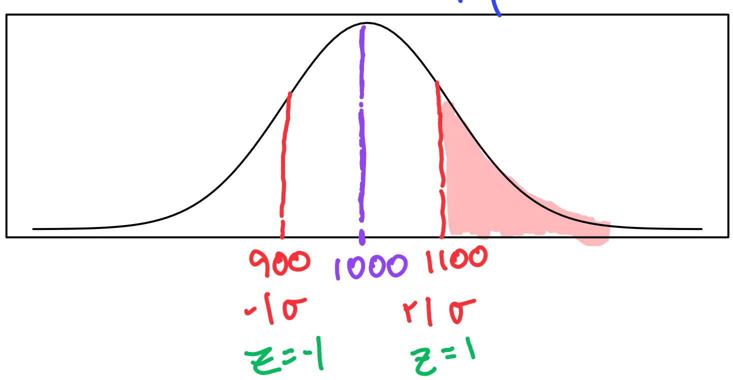
$$P_u - P_L = 0.1498823$$

Problem 2 The life expectancy of a certain type of lightbulb is normally distributed with a mean of 1000 hours and a standard deviation of 100 hours.

1. What proportion of lightbulbs last longer than 1100 hours?

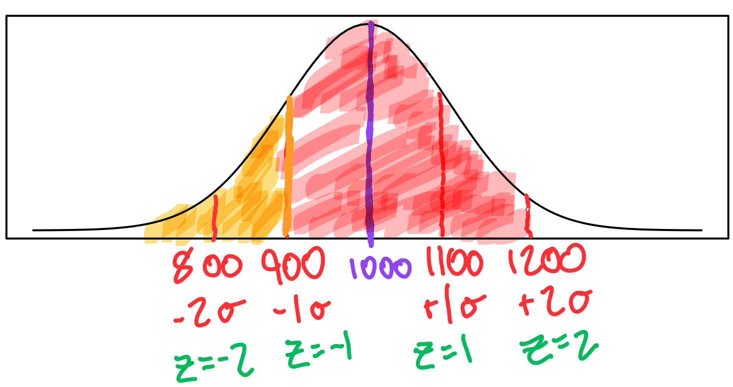
$$Z = \frac{1100 - 1000}{100} = 1.00$$

$$P(Z > 1.00) = 1 - P(Z < 1.00) = 0.1586553$$



2. What proportion of lightbulbs last between 900 and 1200 hours?

$$P(Z < 2.00) - P(Z < -1.00) = 0.8185946$$



3. What proportion of lightbulbs last less than 800 hours?

$$P(Z < -2.00) = 0.02275013$$

Sampling Distributions

Answer the following short-response questions:

1. What is a population?

   THE TOTAL GROUP WE ARE TRYING TO GATHER INFORMATION ON

2. What is a sample?

   A SUBSET OF THE POPULATION THAT IS ACTUALLY OBSERVABLE

3. Which arises from a population, a parameter or a statistic?

   PARAMETER

4. Can we make inference from an entire population?

   NO, AN ENTIRE POPULATION IS IMPOSSIBLE TO OBSERVE, SO WE CANNOT MAKE INFERENCE FROM IT

5. What is the primary goal of statistics?

   TO MAKE INFERENCE ABOUT A POPULATION'S PARAMETERS USING STATISTICS COMPUTED FROM SAMPLES

Problem 1 The average time employees spend on a specific task is normally distributed with a population mean of $\mu = 6$ hours and a standard deviation of $\sigma = 4$ hours. If a random sample of $n = 25$ employees is taken:

1. Find the mean of the sample mean, $\mu_{\bar{x}}$.

   $$\mu_{\bar{x}} = \mu = 6$$

2. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

   $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{5} = 0.8$$

3. Express the distribution of the sample mean $\bar{x}$ in proper notation.

   $$\bar{x} \sim N(6, 0.8^2)$$

Problem 2 The height of a specific plant species has a population mean of $\mu = 17$ cm with a standard deviation of $\sigma = 20$ cm. If a random sample of $n = 100$ plants is selected:

1. Find the mean of the sample mean, $\mu_{\bar{x}}$.

   17

2. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

   $$\frac{20}{\sqrt{100}} = 2$$

3. Express the distribution of the sample mean $\bar{x}$ in proper notation.

   $$\bar{X} \sim N(17, 2^2)$$

Problem 3 A factory produces metal rods with a population mean diameter of $\mu = 50$ mm and a standard deviation of $\sigma = 5$ mm. If a sample of $n = 36$ rods is taken:

1. Find the mean of the sample mean, $\mu_{\bar{x}}$.

   50

2. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

   $$\frac{5}{\sqrt{36}} = \frac{5}{6} \approx 0.833$$

3. Express the distribution of the sample mean $\bar{x}$ in proper notation.

   $$\bar{X} \sim N\left(50, \frac{5}{6}^2\right)$$

4. Find the 33^{rd} percentile of $\bar{x}$.

   $$P(Z < z) = 0.33, \quad z \approx -0.44$$

   $$-0.44 * \frac{5}{6} + 50 = 49.63333$$

Problem 4 The daily sales of a local coffee shop are normally distributed with a population mean of $\mu = 300$ dollars and a standard deviation of $\sigma = 75$ dollars. If a random sample of $n = 49$ days is taken:

1. Find the mean of the sample mean, $\mu_{\bar{x}}$.

   300

2. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

   $$\frac{75}{\sqrt{49}} = 10.714$$

3. Express the distribution of the sample mean $\bar{x}$ in proper notation.

   $$\bar{X} \sim N(300, 10.714^2)$$

4. Find the probability that we will observe a sample mean over 412.

   $$\frac{412 - 300}{10.714} = 10.45361 \qquad P(z > 10.45361) \approx 0$$

   LOL SORRY YALL

5. Find the 91st percentile of $\bar{x}$.

   $$P(Z < z) = 0.91, \; z \approx 1.34$$
   $$1.34 * 10.714 + 300 = 314.3568$$

Problem 5 A car manufacturer tests the gas mileage (in miles per gallon) of its cars. The population mean is $\mu = 25$ miles per gallon, with a standard deviation of $\sigma = 3$ miles per gallon. If a sample of $n = 64$ cars is tested:

1. Find the mean of the sample mean, $\mu_{\bar{x}}$.

   25

2. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

   $$\frac{3}{\sqrt{64}} = 0.375$$

3. Express the distribution of the sample mean $\bar{x}$ in proper notation.

   $$\bar{X} \sim N(25, 0.375^2)$$

4. Find the probability that we will observe a sample mean less than 23.

   ATTEMPT WITH THESE VALUES. AGAIN, SORRY

5. Find the probability that we will observe a sample mean between 23 and 26.

Problem 6

A survey shows that 45% of adults in a city regularly use public transportation. Suppose we take a random sample of $n = 100$ adults.

1. Find the mean of the sample proportion, $\mu_{\hat{p}}$.

$$\mu_{\hat{p}} = p = 0.45$$

2. Find the standard deviation of the sample proportion, $\sigma_{\hat{p}}$.

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.45 * 0.55}{100}} = 0.04975$$

3. Express the distribution of the sample proportion $\hat{p}$ in proper notation.

$$\hat{p} \sim N(0.45, 0.04975^2)$$

Problem 7

It is known that 30% of the population in a certain region prefers organic food. If a random sample of $n = 200$ people is taken:

1. Find the mean of the sample proportion, $\mu_{\hat{p}}$.

$$\mu_{\hat{p}} = p = 0.30$$

2. Find the standard deviation of the sample proportion, $\sigma_{\hat{p}}$.

$$\sigma_{\hat{p}} = 0.032403$$

3. Express the distribution of the sample proportion $\hat{p}$ in proper notation.

$$\hat{p} \sim N(0.30, 0.032403^2)$$