Homework 3 Solutions

1. You read that in native Hawaiians, the probability of having blood type AB is 1/100. What does this mean?

a. If you pick 100 Hawaiians randomly, the fraction of them having blood type AB will be very close to 1/100

b. If you pick 100 Hawaiians randomly, exactly 1 of them will have blood type AB

c. If you pick $10,000$ Hawaiians randomly, exactly 100 of them will have blood type AB

2. A cat is about to have 6 kittens. The sample space for counting th enumber of female kittens she has is

a. S = any number between 0 and 1

b. S = whole numbers 0 to 6

c. S = all sequences of 6 males or females by order of birth, such as FMMFFF

3. $X = \{$the weight of randomly selected cervids$\}$

- Support: $0 < x < \infty$
- Variable Type: Continuous

4. $X = \{$the time it takes to conduct a rare surgery$\}$

- Support: $0 < x < \infty$
- Variable Type: Continuous

(Problem 5) Let Z = the number of owls currently outside your home. The probability distribution of Z is given as follows:

z	0	1	2	3	4	5	6	7+
$P(Z = z)$	0.5	0.25	0.12	0.05	0.03	0.03	0.012	0.008

5a. Is this a legitimate probability distribution?

$$0.5 + 0.25 + 0.12 + 0.05 + 0.03 + 0.03 + 0.012 + 0.008 = 1.0$$

5b. Find the probability that no owls are outside your home.

$$P(Z = 0) = 0.5$$

5c. Find the probability that at least 2 owls are outside your home.

$$P(Z \geq 2) = 0.12 + 0.05 + 0.03 + 0.03 + 0.012 + 0.008 = 0.25$$

5d. Find $\mathbb{E}Z$

$$(0.5 * 0) + (0.25 * 1) + (0.12 * 2) + (0.05 * 3) + (0.03 * 4) + (0.03 * 5) + (0.012 * 6) + (0.008 * 7) = 1.038$$

5e. Find $\mathbb{V}Z$

$$(0.5 * (0 - 1.038)^2) + (0.25 * (1 - 1.038)^2) + (0.12 * (2 - 1.038)^2) + (0.05 * (3 - 1.038)^2) +$$

$$(0.03 * (4 - 1.038)^2) + (0.03 * (5 - 1.038)^2) + (0.012 * (6 - 1.038)^2) + (0.008 * (7 - 1.038)^2)$$

$$= 2.156556$$

5f. Find σ_Z

$$\sqrt{2.156556} = 1.468522$$

5g. At what point are there enough owls outside your home that you're concerned? Find the probability that there are that many or more owls.

At 3 owls, I'm calling someone.

$$P(Z \geq 3) = 0.05 + 0.03 + 0.03 + 0.012 + 0.008 = 0.13$$

(Problem 6)

Let

$$A \equiv \{\text{Your March Madness Bracket is exactly correct}\}$$

$$B \equiv \{\text{Dean Culbertson's Bracket has K-State as the champions}\}$$

$$C \equiv \{\text{K-State isn't in the lineup for 2025}\}$$

Def

$$P(A) = 1/10^{12},\ P(B) = 0.8,\ P(C) = 0.95$$

6a. Find $P(A \cup B)$

It's fair to say that A and B are independent. Thus:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$

$$= 1/10^{12} + 0.8 - 1/10^{12} * 0.8$$

$$= 0.8$$

6b. Find $P(B \cap C)$

It's reasonable to say B and C are mutually exclusive:

$$P(B \cap C) = 0$$

6c. Find $P(A|B)$

Given independence:

$$P(A|B) = P(A) = 1/10^{12}$$

6d. Find $P(B|C)$

Given mutual exclusivity:

$$P(B \cap C) = 0$$

$$P(B|C) = P(B \cap C)/P(C) = 0/0.95 = 0$$