

Biometrics I - Practice Exam 2

1. Identify the possible support and type of each random variable, and assign the *most appropriate* distribution to each

i. $Y = \{\text{Test Preparation Tutoring (Received or Did not)}\}$

- Support: $\{0, 1\}$
- Type: DISCRETE
- Distribution: BERNOULLI

ii. $X = \{\text{Exam Score (0-100)}\}$

- Support: $\{0, 100\}$
- Type: CONTINUOUS
- Distribution: $\frac{\text{SCORE}}{100} \sim \text{BETA}$

2. Let $Q = \{\text{the number of times two dolphins surface for air in a 5-minute observation period}\}$, the distribution of Q is shown below:

q	0	1	2	3	4
$P(Q = q)$	0.15	0.35	0.28	0.14	0.08

a. Prove that this a legitimate probability distribution

$$0.15 + 0.35 + 0.28 + 0.14 + 0.08 = 1.00$$

b. Find $\mathbb{E}W$

$$\begin{aligned} \mathbb{E}W &= \sum w P(W=w) \\ &= 0(0.15) + 1(0.35) + 2(0.28) + 3(0.14) + 4(0.08) \\ &= 1.65 \end{aligned}$$

c. Find $\mathbb{V}W$

$$\begin{aligned} \mathbb{V}W &= \sum (w - \mathbb{E}W)^2 P(W=w) \\ &= ((0 - 1.65)^2(0.15)) + ((1 - 1.65)^2(0.35)) + ((2 - 1.65)^2(0.28)) + ((3 - 1.65)^2(0.14)) + ((4 - 1.65)^2(0.08)) \\ &= 1.2875 \end{aligned}$$

3. Below is the result of a study examining voter registration by age group and political affiliation:

Age Group - Education	Democrat	Republican
18-35 - College Degree	156	87
18-35 - No College Degree	132	108
Over 35 - College Degree	97	123
Over 35 - No College Degree	84	178

a. What's the probability that a person aged 18-35 with a college degree is registered as a Democrat?

$$\frac{156}{965}$$

b. What's the probability that a voter is registered as a Republican, given that they are over 35?

$$\frac{301}{482}$$

4. Determine if events A and B are independent or not given that:

$$P(A) = 0.7, P(B \cap A^c) = 0.12, \text{ and } P(A \cup B) = 0.82$$

$$P(A^c) = 0.3 \quad P(B \cap A^c) = P(B)P(A^c) \quad P(B) = \frac{0.12}{0.30} = 0.4$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.40 - (0.7 * 0.4) = 0.82$$

A & B ARE INDEP

5. Assume A and B are independent. Given $P(B) = 0.6$ and $P(A^c \cap B^c) = 0.16$, find the following:

a. $P(A^c \cup B^c)$

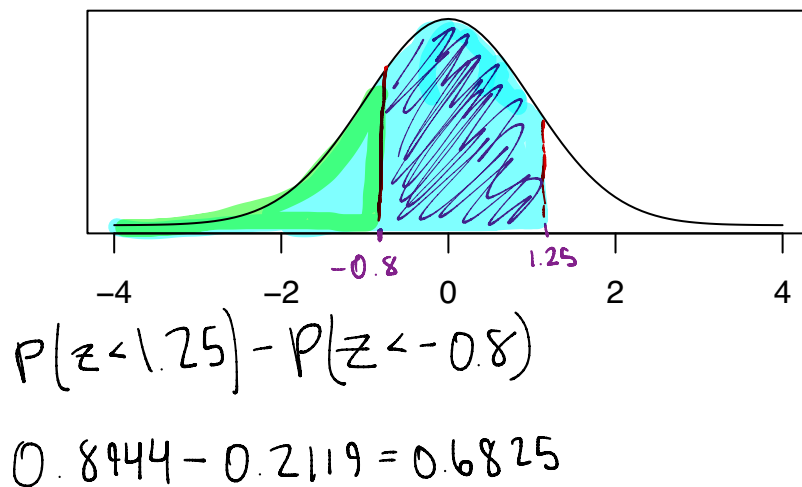
$$P(A^c \cap B^c) = P(A^c)P(B^c) \quad 0.16 = P(A^c)0.4 \quad P(A^c) = \frac{0.16}{0.4} = 0.4$$

$$P(B^c) = 0.4 \quad P(A^c \cup B^c) = 0.4 + 0.4 - 0.16 = 0.64$$

b. $P(B|A) = P(B)$

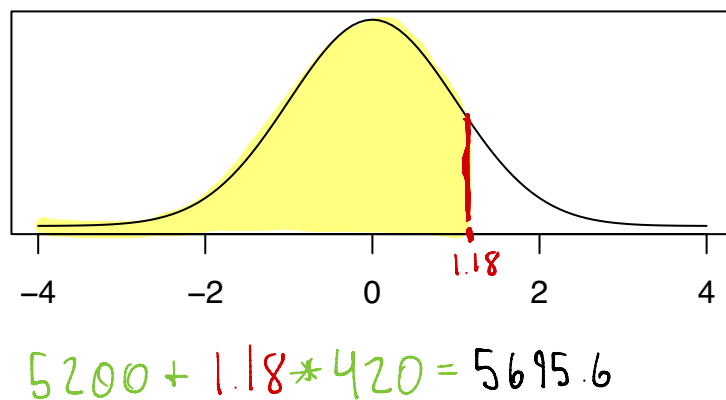
$$= 0.6$$

6. Find $P(-0.8 < Z < 1.25)$

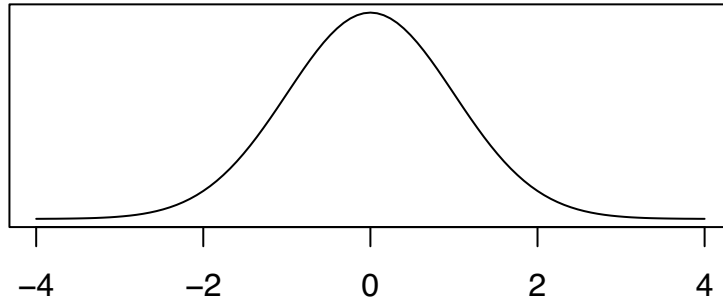


7. The average weight of male elephants is 5200 kg with a standard deviation of 420 kg. The average weight of female elephants is 3500 kg with a standard deviation of 280 kg.

a. What weight is considered 88th percentile for male elephants?



- b. What is the probability that a female elephant is under 3300 kg?



$$z = \frac{3300 - 3500}{280} = \frac{-200}{280} = -0.71$$

$$P(z < -0.71) = 0.2420$$

8. The daily visitors to a national park are normally distributed with a population mean of $\mu = 1200$ visitors and a standard deviation of $\sigma = 175$ visitors. If a random sample of $n = 18$ days is taken:

- a. Find the mean of the sample mean, $\mu_{\bar{x}}$.

$$\mu_{\bar{x}} = \mu = 1200$$

- b. Find the standard deviation of the sample mean, $\sigma_{\bar{x}}$.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{175}{\sqrt{18}} = 41.25$$

- c. Express the distribution of the sample mean $\bar{x}$ in proper notation.

$$\bar{X} \sim N(1200, 41.25^2)$$

$$\bar{X} \sim N(1200, \frac{175^2}{18})$$

9. An experiment is conducted to observe the difference between growth rates of two varieties of corn. Each variety is planted in a plot that will receive high fertilizer, and a plot that will receive low fertilizer. The plants will be measured after 45 days.

a. What is/are the EU?

PLOTS

b. Trt(s)?

H/L FERTILIZER

c. Response?

GROWTH RATE

d. Factors?

CORN VARIETIES

e. Number of EU per Trt?

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